BRTS
Bamford Rock Testing Services

LABORATORY TESTS FOR STRENGTH, HARDNESS AND ABRASIVENESS : EXPLANATORY NOTES

Sklerograf Hardness :
This is measured using a Harteprufer SKLEROGRAF Modell D instrument supplied by Roell & Korthaus KG.
The reported value is the mean of 5 spot readings on the smooth polished end of a cylindrical core, clamped in a heavy V-block, with its axis vertical.
One reading is taken at the centre and 4 readings are taken at points midway between the centre and the outside edge of the core, along 4 quadrant lines.

Coefficient of Plasticity K :
The core under test is held rigidly vertical, and the Sklerograf Hardness tester is also held fixed on the axis of the core. 20 impacts of the Sklerograf Hardness tester are made on the same point, and the mean of readings 11 to 20 is taken as the "final hardness value"
The Coefficient of Plasticity K is calculated as

$$\frac{(\text{Final Hardness Value}) - (\text{Initial Hardness}) \ast 100\%}{(\text{Final Hardness Value})}$$

Shore Hardness :
Not directly measured, but deduced from the Sklerograf Hardness, using data supplied by Roell & Korthaus KG.

Brinell Hardness :
Not directly measured, but deduced from the Sklerograf Hardness, using data supplied by Roell & Korthaus KG.

Rockwell Hardness :
Measured with a HR-150AI Hardness Tester supplied by Laizhou Hengguan Testing Instruments, whose measurement ranges are 20-95HRA, 10-100HRBW, or 20-702HRC.

Brinell and Rockwell Hardness are standard metallurgical test values, so measuring the equivalent values for a rock gives a useful indication of the minimum hardness which any metal surface likely to come into contact with a particular rock should have, in order to resist suffering abrasive wear by that rock.

Schmidt Hammer Hardness :
Measured with a RockSchmidt Type L, made by Proceq, having impact energy of 0.735Nm.

Moisture Content / Porosity :
A fragment of rock (usually weighing at least 200 grams) is oven-dried at 105°C for 48 hours, and weighed.
It is then fully saturated, by being immersed in water in a sealed container, which is connected to a vacuum pump which applies a partial vacuum, of the order of 60 kPa, to the air above the water, for a period of 48 hours, then re-weighed.
The mass difference allows the fully-saturated moisture content to be calculated. The porosity of the rock may be inferred as being equivalent to the fully-saturated moisture content.

Wet Density :
Calculated by dividing the mass (measured on an electronic balance to a precision of 0.01 grams) by the volume of the fully-saturated uniaxial compression test specimen ($\pi l d^2/4$), where the length l and the diameter d are both measured by vernier calipers, to a precision of 0.1 mm

Dry Density :
Calculated by dividing the Wet Density by {1 + Moisture Content(%)/100}
OR
Calculated by dividing the mass (measured on an electronic balance to a precision of 0.01 grams) by the volume of the oven-dried uniaxial compression test specimen ($\pi l d^2/4$), where the length l and the diameter d are both measured by vernier calipers, to a precision of 0.1 mm

Dynamic Elastic constants :

The velocities with which elastic waves travel through an elastic solid are functions of the elastic modulus E, the Poisson's ratio ν, and the density ρ

Compressional wave velocity $V_P = \sqrt{(E(1-\nu) / (\rho(1+\nu)(1-2\nu)))}$

Shear wave velocity $V_S = \sqrt{(G/\rho)}$

Modulus of rigidity $G = E/2(1 + \nu)$

A Terrametrics pulse generator sends 1000V pulses at 100 c.p.s into Compressional Wave and Shear Wave piezolectric transducer heads.

Measurement with a Hewlett Packard cathode-ray oscilloscope of the travel times of the ultrasonic pulses through a cylinder of rock enables calculation of the compressional and shear wave velocities (= specimen length/measured travel times), and thence of dynamic E, ν, G and K .

Elastic modulus $E = \rho V_S^2(3 V_P^2 - 4 V_S^2)/(V_P^2 - V_S^2)$

Poisson's ratio $= (V_P^2 - 2V_S^2) /2 (V_P^2 - V_S^2)$

Modulus of rigidity, or Shear modulus $G = \rho V_S^2$

Bulk modulus $K = \rho (3V_P^2 - 4V_S^2)/3$

Transmitted Amplitude Ratio (T.A.R.) :

A semi-quantitative indication of the soundness of the rock, the Transmitted Amplitude Ratio (T.A.R.) is expressed as the ratio of the amplitudes of the shear waves, measured on the screen of a cathode ray oscilloscope, received after being transmitted through 2 cylinders having the same dimensions - one cylinder being the tested rock, the other being a standard metal.

The standard metal is free-machining Aluminium alloy 2011 (5.5% Cu, 0.5% Bi, 0.5% Pb). This alloy has an elastic modulus of 70 GPa, and characteristic P-wave and S-wave velocities of 6100 and 3100 metres/sec respectively. These properties are closer than those of steel to those of most rocks, so making Al a better calibration material than steel.

The 2 S-wave transducer heads are placed in contact with the ends of the Al cylinder and the amplitude of the S-wave transmitted through the cylinder is displayed on the screen of the C.R.O. (e.g. 52 millivolts). The test sample is placed between the S-wave transducers and the amplitude of the S-waves transmitted through the sample is measured on the screen of the C.R.O. (e.g. 15 mv for a sample of weathered Silurian mudstone).

The T.A.R. would be expressed as 15/52 = 0.29

The lower the ratio is for a test specimen, the greater is the degree of microcracking and microfissuring likely to be present within the specimen.

Tensile Strength :

Usually measured by the indirect (or “Brazilian”) method : $T_o = 2F/\pi dt$

This can also be done by the direct pull method, if required, as suitable gripping jigs for use in a direct tension machine have been constructed here.

Fracture Toughness :

A measure of the stress intensity required to initiate crack propagation. The Short Rod Chevron Notch (SR) method is performed on cores less than 55mm diameter, and the Cracked Chevron Notch Brazilian Disc (CCNBD) method is performed on cores greater than 50mm diameter. The tests measure the resistance of the rock to being “pulled apart” over a very small cross-sectional area – the tip of a V-notch – and so effectively the intrinsic tensile strength of intact rock substance.

Critical Energy Release Rate :

Or critical crack driving force - The fracture material property which is a measure of the energy required to create new surface area; a function of the fracture toughness, Poisson’s ratio, and the modulus of elasticity (i.e. both rock strength and stiffness). G_{Ic} has units N/m

Field Penetration Index :

Nelson, Ingraffea & O'Rourke studied TBM performance data, and correlated the penetration per revolution and the Field penetration index, R_f (kN/mm) with fracture toughness.

Specific energy of cutting :

Specific energy of cutting by roadheaders may be calculated from the fracture toughness, using correlations published by Fowell (1991)

Modulus of Rupture :

This is the calculated bending stress at failure of a specimen under three-point load – the "outer fibre tensile strength". The standard specimen size is a rectangular prism 200mm by 100mm by 60mm. The loads are applied through 3 parallel steel rods having a diameter of 25mm; one rod at the centre of the upper face, the other 2, spaced 180mm apart, supporting the lower face. $T = 3WL/2BD^2$

Where T = Modulus of Rupture (Mpa); W = Load at failure (N); L = Span length (=180mm);
B = Width of specimen (≈100 mm); D = Thickness of specimen (≈60 mm)

Shear Strength :

Measured by the punch shear method, whereby a central core is punched through the remaining annulus of a thin disc of rock, held confined in a punch and die apparatus.

Compressive Strength Tests

The standard conditions for the preparation of specimens for compressive testing, suggested by the International Society for Rock Mechanics (I.S.R.M.) are :
Diameter preferably 54 mm, or greater;
Diameter preferably at least 10 times the size of the largest grain in the rock;
Length to diameter ratio = 2.5 to 3.0
The ends of the specimen must be parallel to each other, and at right angles to the longitudinal axis of the core cylinder;
The ends must be ground flat, with a maximum allowable surface relief of 0.02 mm;
The ends must be perpendicular to the axis to within 0.001 radian, or 0.05 mm in 50 mm;
The sides must be smooth, and straight to within 0.3 mm over the full length of the specimen;
The specimens should be either saturated under a vacuum for 48 hours, or oven-dried for 48 hours before testing (preferably 2 specimens should be tested - one saturated and one dry, from every sample);
The loading rate should be no greater than 1 MPa/second, and should produce compressive failure within 5 to 10 minutes of the commencement of loading.
The platens should be hardened and ground steel, and the specimens are polished, to reduce the friction between rock and platens. If platens and specimens have different elastic moduli and Poisson's ratios there will be differential expansions under load;
if there is appreciable friction between the platens and the rock, the effect on the stress distribution within the testspecimen near its ends will depend upon whether the platen is softer or stiffer than the rock.
If the platen is softer (i.e. if a plaster or gypsum capping is used, to avoid having to grind the ends of the specimen), the capping will try to expand more than the specimen, applying radial tensile stresses to it, and possibly splitting it.
If the platen is stiffer (as steel is stiffer than most rocks) the specimen will try to expand more than the platens, and be restrained by them - the specimen will be in a state of triaxial stress, rather than uniaxial stress.
This end effect is significant only within a longitudinal distance equivalent to less than half the specimen diameter, so that if the specimen is as long as is recommended the majority of it will be under the assumed uniaxial loading conditions.

Uniaxial Compression Test

Determination of elastic modulus and Poisson's ratio during the uniaxial compression can be made from measurements of axial deformations or strains, and either radial deformations or circumferential strains.
Because of the inherent natural variability of rocks it is necessary for a number of replicate tests to be performed.
The minimum number of tests is usually at least 5, although it has been shown that 10 tests would be required to be fully confident of knowing the properties of a material with a coefficient of variation (*i.e. Standard Deviation/Mean*) of 20%, and a correspondingly higher number of tests for a more variable material.
The design values of strength and deformability should be determined from a knowledge of the distribution of values that has been found (usually a standard normal distribution or a log-normal distribution can be fitted to rock properties), and the acceptable risk of failure.
e.g. if the compression testing found that the test values could be described by a standard normal distribution, with a coefficient of variation of 20%, and it was decided that a probability of failure of 5% could be tolerated,
then the design strength would be taken as the Mean Strength - 1.645 Standard Deviations,
or Design Strength = Mean Strength(1 - 1.645*0.2) = 0.671*Mean Strength,
or the "Required Factor of Safety" = 1/0.671 = 1.49 (the Mean Strength must be 1.49 times the Design Stress value).

Uniaxial Compressive Strength :

The maximum axial load sustained by a specimen, at the point of failure, is recorded, to a precision of 0.5 kiloNewton (if the load is greater than 90 kiloNewtons) or to a precision of 100 Newtons (if the load is less than 90 kiloNewtons). This load is divided by the mid-height cross-sectional area of the specimen at the point of failure, as measured by radial deformation transducers, to give the failure stress in MegaNewtons per square metre, or MPa.

Uniaxial Compressive Strength :	*I.S.R.M. Standard Terminology*
< 1 MPa	" *Extremely low strength* "
1 - 5 MPa	" *Very low strength* "
5 - 25 MPa	" *Low strength* "
25 - 50 MPa	" *Medium strength* "
50 - 100 MPa	" *High strength* "
100 - 250 MPa	" Very *high strength* "
> 250 MPa	" *Extremely high strength* "

Ratio of Soaked to Dry Strength

This ratio indicates the change in strength when a rock is soaked with water, and is relevant to performance under conditions where a rock or building stone is affected by rain, by rising or falling damp, or in wet situations such as tunnels or sea walls. The ratio may be an important factor in design where it is less than 0.5, as in some argillaceous sandstones and porous limestones. It is an important in the design of thin claddings, and of tunnels constructed below the water table, where the soaked strength should be used in calculations.

Static Elastic Constants :

Axial deformations of the compression test specimen are measured by 2 L.V.D.T.s, one mounted, parallel to the axis, on each side of the specimen; the average of the 2 readings indicates the average deformation along the core cylinder axis.

Radial deformations of the compression test specimen are measured by 3 L.V.D.T.s, mounted 120^o apart in a plane perpendicular to the core cylinder axis, at the mid-height of the specimen; the average of the 3 readings indicates the average radial deformation of the core cylinder at its midpoint.

Static Secant E :

On the plot of axial stress versus axial strain the slope of the least-squares regression straight line, from the onset of loading up to an axial stress equivalent to half the maximum axial stress sustained by a specimen at the point of failure, is the secant Young's Modulus (E).

Mid-third E :

On the plot of axial stress versus axial strain the slope of the least-squares regression straight line, from an axial stress equivalent to one-third of the maximum axial stress sustained by a specimen at the point of failure up to an axial stress equivalent to two-thirds of the maximum axial load sustained by a specimen at the point of failure, is the mid-third Young's Modulus (E).

Secant Poisson's Ratio :

On the plot of axial stress versus axial and radial strains, from the onset of loading up to an axial stress equivalent to half the maximum axial stress sustained by a specimen at the point of failure, the slope of the least-squares regression straight line fitted through the measured average radial strain is divided by the slope of the least-squares regression straight line fitted through the measured axial strain, to calculate the secant Poisson's Ratio.

Mid-third Poisson's Ratio :

On the plot of axial stress versus axial and radial strains, from an axial stress equivalent to one-third of the maximum axial load sustained by a specimen at the point of failure up to an axial load equivalent to two-thirds of the maximum axial load sustained by a specimen at the point of failure, the slope of the least-squares regression straight line fitted through the measured average radial strain is divided by the slope of the least-squares regression straight line fitted through the measured axial strain, to calculate the mid-third Poisson's Ratio.

Modulus Ratio

$$= \frac{\text{Young's Modulus } E}{\text{Uniaxial Compressive Strength } C_o}$$

< 200	" *Low Modulus Ratio* "
200 - 500	" *Normal Modulus Ratio* "
> 500	" *High Modulus Ratio* "

Critical Energy Release Rate & Field Penetration Rate :
The Critical Energy Release Rate, in N/m, may be calculated from the Fracture Toughness, Young's Modulus and Poisson's ratio.
The Field Penetration Rate for tunnel boring machine disc cutters, R_f (N/mm) may be calculated from the Critical Energy Release Rate, using correlations published by Nelson, Ingraffea & O'Rourke (1985)

Angle of Shearing Resistance, Φ :
(i) Inferred as the slope of the line, on a τ vs σ plot, passing through an intercept on the τ axis, $S_0 = 2\ T_0$ (as per the Griffith criterion), tangent to the Mohr stress circle for the unconfined compressive strength.

(ii) $\Phi = 90^0 - 2\alpha$, where α = the angle between the core axis and the plane of an observed shear failure.

Uniaxial Compressive/Tensile Strength Ratio
This ratio is an indicator of the toughness of a rock, and is of fundamental importance in assessing cuttability by roadheader or tunnel boring machine.
It has also been found to be approximately equal to "m" in the Hoek-Brown rock failure criterion.

Brittleness Index B_1 = Uniaxial Compressive Strength / Uniaxial Tensile Strength
(= m)

Brittleness Index B_2 = $\frac{\text{Uniaxial Compressive Strength - Uniaxial Tensile Strength}}{\text{Uniaxial Compressive Strength + Uniaxial Tensile Strength}}$
(= sin ϕ)

Brittleness Index B_3 = (Uniaxial Compressive Strength * Uniaxial Tensile Strength)/2 (MPa^2)

Brittleness Index B_4 **= $\sqrt{B_3}$** (MPa)

Altindag - Rock Mech Rock Eng (2010) 43 : 361-370

Brittleness Index $B_{U\text{-}I}$ **= U_e / U_{total}**
where U_e = elastic strain energy at peak stress
and U_{total} = total strain energy, measured up to the point where the post-failure stress has dropped to one third of the peak stress value.

Munoz, Taheri, Chanda - Rock Mech Rock Eng (2016) 49 : 343-3355

Specific Energy (Strain Energy at Failure)
The energy (expressed in kiloJoules per cubic metre) absorbed by a uniaxial compressive strength test specimen, up to the point of its strength failure (i.e. where the stress/strain curve reaches its peak value, and then takes a negative slope). Obtained by measuring the area under the Axial Stress versus Axial Strain curve, from commencement of loading, until peak stress value is reached.

Analysis of results of past testing in this laboratory shows the following distribution of Specific Energy values:

Lower decile	41
Lower quartile	76
Median	176
Upper quartile	314
Upper decile	592

Specific Energy (Specific Energy of Destruction)
The energy (expressed in kiloJoules per cubic metre) absorbed by a uniaxial compressive strength test specimen, from the start of the test, beyond the point of its strength failure, until it has lost all significant strength.
Obtained by measuring the area under the complete Axial Stress versus Axial Strain curve.

Maximum Distortional Strain Energy

The maximum-distortion-energy theory of failure partitions strain energy into a component causing volume change (without distortion) and a component causing distortion.
Only the latter component will cause inelastic behaviour : yielding or fracture.

It is a function of the measured Uniaxial Compressive Strength, Young's Modulus, and Poisson's Ratio.

$$(u_d)_f = \frac{(1 + \nu)}{3E} \; . \; \sigma_f^2$$

Rock Toughness Index

= Strain Energy At Failure (kJ/m^3)/ Uniaxial Compressive Strength (MPa)

Rock with a "Normal" Modulus Ratio should have a Rock Toughness Index between 1 and 2.5

e.g. linear elastic behaviour, with no plastic deformation before failure :
Modulus Ratio = 200 ⇒ Rock Toughness Index= 2.5
Modulus Ratio = 350 ⇒ Rock Toughness Index = 1.43
Modulus Ratio = 500 ⇒ Rock Toughness Index = 1.0

Rock Toughness Index values of less than 1.0 may indicate brittle, easily broken rocks.
Rock Toughness Index values of greater than 2.5 may indicate tough, difficult-to-cut rocks, and/or rocks which may store abnormally high levels of strain energy before failure, and so be prone to rock-bursting.

Bamford - IX Australian Tunnelling Conference, Sydney (1996) 215-221

Fracture Energy

The measured Axial Force at failure multiplied by the measured compressive deformation.
Standardized for a test specimen 50mm diameter by 50mm long.

Specific Fracture Energy

Fracture Energy divided by Uniaxial Compressive Strength
Standardized for a test specimen 50mm diameter by 50mm long.

The following section is courtesy of Mr. Max Lee of AMC Consultants, and reflects his original ideas

Energy Consumption per Crack Area

=[(Fracture Toughness)^2 $(1-\nu_s^2)$]/E_s
It is a measure of the amount of energy consumed when propagating a crack per unit area.
Rocks that have a low energy consumption per crack area will generally produce large areas of new cracks at failure.
They are also likely to behave in a brittle manner, particularly if they are igneous, siliceous and/or calcite rich.
In contrast, “soft” ultramafic rocks tend to have high energy consumptions per crack area and typically tend to behave in a ductile manner.

Crack Length

=(Strain Energy at Failure/Energy Density per Crack Area)
A measure of the total “length” of crack that can be developed/propagated at failure.
Rock types that have high crack “lengths” tend to exhibit extensive high stress “onion” slabbing.

Crack Potential

=(Crack length)/(Fracture Toughness)
This index is intended to show which rock types are more likely to produce more crack area at failure, and are likely to behave in a brittle manner.

Strain Burst Index

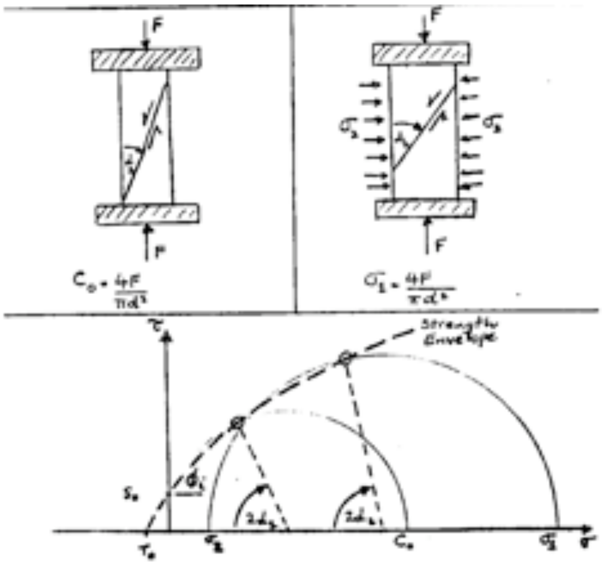
=(E_D/E_S)/(Crack potential)
This index is intended to highlight rocks that have a tendency to fail early and violently.
Rocks having low E_D/E_S ratios are considered to have more microcracks, pores and “thick” grain boundaries, on which early failure is likely to initiate. These imperfections are also likely to assist the rapid propagation of cracks. It is therefore argued that these rocks will be more prone to violent cracking.
Rocks that have high crack potentials produce more crack area, at failure, and also tend to behave in a brittle manner.
A low strain burst index suggests that a particular rock type has a high potential to be strain-burst prone when it is exposed around underground openings.

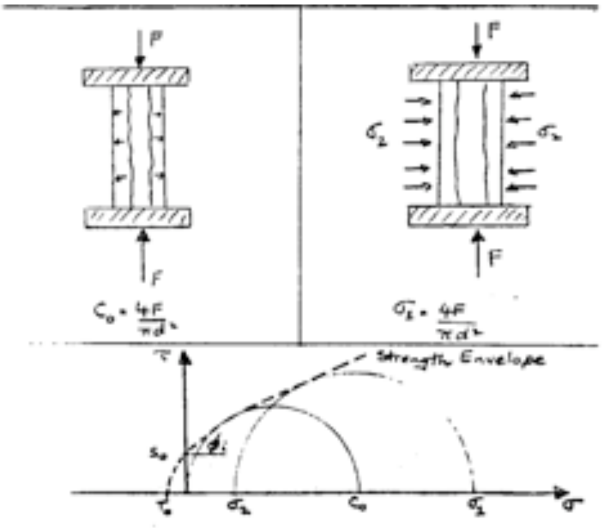
General Predictions :

Strain Burst Index > 1.0 : Rocks tend to behave in a ductile, non-violent manner.
Strain Burst Index < 1.0 : Rocks tend to behave in a brittle, violent manner.
(Strain Burst Index < 1.0, and $E_D/E_S < 1.0$: Rocks tend to exhibit High Stress Slabbing.)
Strain Burst Index < 0.5 : Rocks tend to be Strain Burst Prone.

Triaxial Compression Test

The test specimen is enclosed in a rubber or adiprene jacket, to prevent penetration of the hydraulic fluid into the pores, placed in a heavy cylindrical vessel or "triaxial cell" and subjected to a constant hydrostatic confining pressure σ_3 and a constantly increasing axial load until failure is imminent.
If the confining pressure is then rapidly increased a second and third Mohr stress circle representing "peak strength" conditions can be determined.
If the specimen is then allowed to fail, and the confining pressure is reduced after each failure event, several Mohr stress circles representing the "residual strength" of the rock, or the shear strength of the failure plane can be determined.
The peak strength failure envelope is usually drawn tangent to the Mohr circles representing conditions at the onset of each failure, and is usually found to be either a straight line or a concave-downwards curve, parabolic in shape.
The residual strength failure envelope is plotted after measuring the angle α between the failure plane and the axis of the specimen.
The shear and normal stress on this plane are calculated or determined graphically, and the residual failure envelope is the line of best fit through these points.
In this way the peak strength parameters S_0 and ϕ_i and the residual strength parameters c_r and ϕ_r can be determined.





References :
"Standard Method Of Test For Multistage Triaxial Strength Of Undrained Rock Core Specimens Without Pore Pressure Measurements" - U.S. Army Corps Of Engineers, WES, RTH 204-80
"Standard Test Method for Triaxial Compressive Strength of Undrained Rock Core Specimens Without Pore Pressure Measurements" - ASTM D 2664-95a

Goodrich Test

This test was originally suggested by Ross Goodrich, of the Joy Manufacturing Co., using similar apparatus to that used in the already-established Sievers test.
The mutual damage done by a tungsten carbide microbit (3/8" or 9.5mm wide, with a 90° included angle) to the rock, and by the rock to the microbit, are measured.

Goodrich Drillability is found from the measured depth of a hole drilled under a standard thrust, by 150 revolutions of a standard tungsten carbide rotary bit. (Low values indicate tough rocks, high values indicate soft rocks.)
Goodrich Wear Number is found from the measured width of the wear flat on the bit used for the Goodrich Drillability test.
(Low values indicate non-abrasive rocks, high values indicate highly abrasive rocks.)

The ratio of Goodrich Drillability to Goodrich Wear Number is correlated with the ability of a roadheader to economically cut a rock.
Ratios of greater than 10 indicate that the rock should be economically cuttable by a light machine such as an AM50 or a Mitsui-Miike S125.
Ratios of greater than 5 indicate that the rock should be economically cuttable by a medium machine such as an AM75 or a Mitsui-Miike S200.
Ratios of greater than 2 indicate that the rock should be economically cuttable by a heavy machine such as an AM105 or a Mitsui-Miike S300.

Sievers J-Value :

Uses the same test apparatus as the Goodrich Drillability Test, but the microbit has slightly different geometry (a 110° included angle), and the test specimen is subjected to 200 revolutions of the microbit (rather than 150).

Taber Abraser test :

The mutual damage done by a carborundum (silicon carbide) disc to a disc of rock, and by the rock disc to the carborundum disc, are measured.

Taber Abradability is found from the measured loss of mass of the rock disc after it has rotated for 800 revolutions under a standard-loaded carborundum disc. (Low values indicate tough rocks, high values indicate soft rocks.)
Analysis of past decades' results of testing in this laboratory shows the following distribution of Taber Abradability values:

Lower decile	0.39
Lower quartile	4.3
Median	47.5
Upper quartile	878
Upper decile	9615

Taber Abrasiveness is found from the measured loss of mass of the carborundum disc.
(Low values indicate non-abrasive rocks, high values indicate highly abrasive rocks.)
Analysis of past decades' results of testing in this laboratory shows the following distribution of Taber Abrasiveness values:

Lower decile	0.16
Lower quartile	5.7
Median	183
Upper quartile	15586
Upper decile	551652

Abrasive Wear Index, as standardised by A.S.T.M. C-501, is calculated as 88/(Rock Disc Loss : grams)
Index of Abrasion Resistance I_w, as standardised by A.S.T.M. C-1353, is calculated as
(36.75/(Rock Disc Loss : grams)) * (bulk density : grams/cc) * (number of revolutions/1000)

Dr. Peter Tarkoy developed a method using the same laboratory test procedure, but defining Abrasion Hardness H_A as 1/(rock disc weight loss) and Rock Abrasiveness A_R as 1/(carborundum disc weight loss).
“Total Hardness” is defined as $H_R \sqrt{H_A}$ (gms$^{-1/2}$) where H_R = Schmidt Hammer hardness.

“Total Hardness” has been correlated with, and may be used as a predictor of, the rate of advance of tunnel boring machines.

Coefficient of Rock Strength (C.R.S.) :
This is the variant, standardised by Paone and Tandanand at the U.S. Bureau of Mines, of the Protodyakonov shatter strength test.
It measures comminution resistance, in terms of energy/unit volume.
Each sample consists of 2 chips or irregular lumps of rock, passing a 25.4mm screen, retained on a 19.1mm screen.
The proportion reduced to -0.5mm size after an arbitrary number of drops of a standard 2.4 kg weight falling 635mm is used to calculate the Coefficient of Rock Strength. The tests are repeated on different samples of the same rock, with different numbers of drops, to find the minimum calculated value, which is reported as the CRS.
CRS, together with operating air pressure, is used to calculate the penetration rate of percussive drills.

Rock Impact Hardness Number (R.I.H.N.) :
This is the variant, standardised by Dr. N. Brook of Leeds University, of the Protodyakonov shatter strength test.
It measures comminution resistance, in terms of the amount of energy to produce an arbitrary proportion of fines.
Each sample consists of a cylinder of rock with a volume of 25.4 cubic centimetres.
The Rock Impact Hardness Number is the number of drops of a standard 2.4 kg weight falling 635mm which results in 25% of the original mass passing through a 0.5mm screen.

RIHN, together with Shore Hardness and operating air pressure, may be used to calculate the penetration rate of down-the-hole hammer percussive drills.

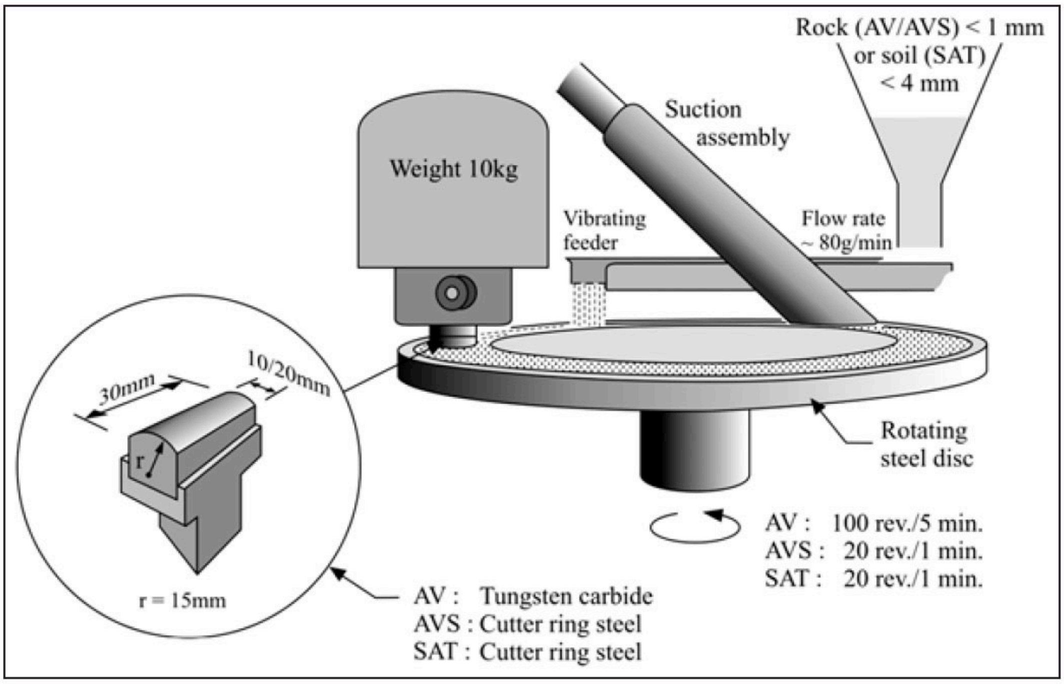
Impact Strength Index (I.S.I.) :
This is the variant, standardised by Ivor Evans & C.D. Pomeroy at the UK Mining Research Establishment, of the Protodyakonov shatter strength test. It measures crushability.
A 100 g sample of crushed rock, in the size range 3.18 – 9.52 mm, is oven-dried at 110°C for 48 hours and then is placed inside a cylinder of 42.86 mm diameter and a 1.8 kg weight is dropped 20 times from a height of 30.48 cm onto the rock sample.
The mass of material remaining in the original size range, expressed as a percentage of the original mass, is the Impact Strength Index (I.S.I.) value.
It is correlated with the Crushability Index of rocks, which measures the comminution produced by a jaw crusher.

Swedish Brittleness Test (S.B.N.) :
Another measure of resistance to comminution.
Each sample consists of 500 grams of rock fragments, passing through a 16mm screen, and retained on a 11.2mm screen. Energy is passed into the rock fragments by a 14 kilogram weight falling 250mm, 20 times.
The percentage of the original mass which then passes through the 11.2mm screen is reported as the friability value or the Swedish Brittleness Number S_{20}

The Norwegian Abrasion test (NAV/AVS/SAT)

A methodology for estimating the drillability of rocks by percussive drilling was developed at the Engineering Geology Laboratory of the Norwegian Institute of Technology (NIH) in 1961.
Abrasion testing of crushed rock particles <1 mm, as illustrated in the Figure below, was then introduced together with the Swedish Brittleness test and the Sievers-J miniature drill test for estimating the drillability parameters DRI (Drilling rate index) and BWI (Bit Wear Index).

Since the early 1980's, the tests have been used mainly for predicting hard rock TBM wear performance according to the method developed by the NTH/NTNU Department of Building and Construction Engineering (in 1996, as result of a merger, NIH changed name to NTNU - the Norwegian University of Science and Technology – and the Norwegian method now is referred to as the NTNU method).



Principle sketch of the NTNU abrasion tests

For TBM cutter wear prediction, a test piece of cutter steel is used instead of the tungsten carbide test piece used for percussive drilling estimation, and the parameter CLI (cutter life index) is calculated instead of BWI.

The Abrasion Values NAV/AVS represent time dependent abrasion of tungsten carbide / cutter steel caused by crushed rock powder. The same test equipment as for the NAV is used to measure the AVS, but instead of the tungsten carbide test pieces used for NAV, the AVS test uses test pieces of steel taken from a cutter ring.

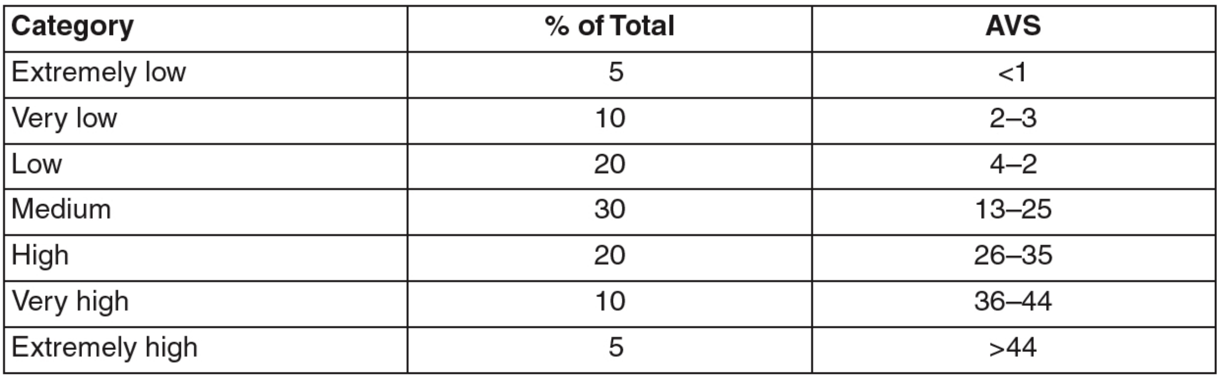
The two tests are defined as follows:
NAV: The Norwegian Abrasion Value is the mean value of the measured weight loss in milligrams of 2 -4 tungsten carbide test work-pieces after 5 minutes, i.e. 100 revolutions of testing, by using an abrasion apparatus and crushed rock powder.
AVS: As described for NAV, but with 1 minute, i.e. 20 revolutions of testing.

For the AVS-test, the standard NTNU / SINTEF test procedure (shown in the figure above), is as follows:

- A representative rock sample consisting of approx. 2kg is used for preparation of abrasion powder.
- Crushing is done gently in several crusher steps to avoid excessive production of fines. The initial crushing is performed in a jaw crusher with the outlet opening adjusted to 10mm. Further crushing is performed using a smaller laboratory crusher in minimum 2 steps. The outlet opening is adjusted to approx 3mm-4mm prior to the first crusher step.
- The crushed material is sieved on a 1 mm quadratic mesh. The fraction < 1 mm is transferred to a suitable pan and the fraction > 1 mm is crushed again after adjustment of the outlet opening to approx. 1 mm. This process is repeated until the grain size distribution is 99% <1 mm and 70±50/ <0.5mm.
- The crushed powder is mixed thoroughly before pouring it into the funnel on the vibrating feeder connected to the abrasion apparatus. The test apparatus is set-up by starting the rotation of the steel disc together with the suction assembly and gradually adjusting the vibrating feeder until a thin and uniform layer of abrasion powder covers the track.
- 2-4 cutter steel test work-pieces are prepared by grinding them to the specified dimensions. The grinding of the test surface is a critical step and extra care is needed to avoid overheating. The edges of the test surfaces are ground, honed and visually examined to make sure that they are smooth and straight. The test bits must also be absolutely clean and dry before weighing to the nearest 0.001g.
- One of the controlled test pieces is clamped under the load and placed gently on the steel disc. The test surface should be horizontally aligned with the steel disc, as it should otherwise be adjusted by the clamping of the test piece and the suspension of the load.
- Testing time is 1 minute, i.e. 20 revolutions. The amount of abrasion powder fed onto the steel disc should be sufficient, but not excessive. It is therefore important to adjust the vibrating feeder during the test in order to avoid steel against steel abrasion or a pile of powder in front of the test piece. The operator should also make sure that the test piece runs in the middle of the track and that a single point of it does not bear directly against the steel disc.
- Test pieces from 2-4 parallel tests are rinsed and dried thoroughly before weighing. The weight loss is calculated, and the results should normally not deviate by more than 5 units (mg).

The NAV or AVS is reported in units of micrograms of mass lost from the work-piece.

AVS classification for rocks based on the NTNU/SINTEF database of 1590 rock samples



Category	% of Total	AVS
Extremely low	5	<1
Very low	10	2–3
Low	20	4–2
Medium	30	13–25
High	20	26–35
Very high	10	36–44
Extremely high	5	>44

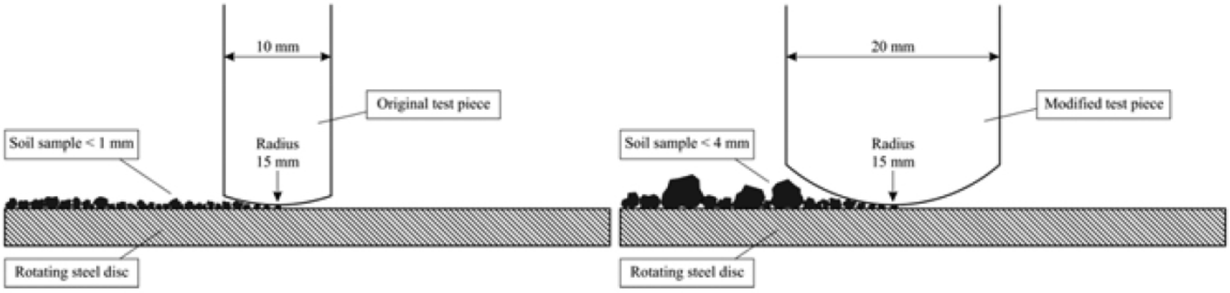
Analysis of past decades' results of testing in this laboratory shows the following distribution of Abrasion Value Steel values:

Lower decile	0.5
Lower quartile	1.6
Median	13.2
Upper quartile	23.9
Upper decile	81.3

The Soil Abrasion Test (SAT)

The new NTNU Soil Abrasion Test is a further development of the earlier abrasion tests for rock. Compared with the AVS test, only two details were changed :

- Instead of crushed rock powder < 1 mm a sieved soil sample with an upper grain size limit of 4mm is used in the SAT test
- the geometry of the original test pieces, as used for NAV & AVS were modified (enlarged) for the SAT



The original (left) and modified SAT test pieces (right)

Soil samples are dried gently in a ventilated oven at 30°C for 2-3 days. After drying the following techniques are used in order to disintegrate and separate the particles for the abrasion powder :

1. Disintegration by use of a soft hammer.
2. Initial disintegration with a jaw crusher of any very hard lumps of cohesive material.
3. Sieving.

SAT-testing of the sieved fraction is carried out according to the same procedures as for AVS-testing.

Drilling Rate Index DRI :

Found as a function of Swedish Brittleness Number, S_{20} and Sievers J-Value.

DRI, together with air pressure, is used to predict the penetration rate of percussion drills.

DRI is also one of the input parameters into the NTU-SINTEF method of predicting tunnel boring machine performance.

Bit Wear Index BWI :
Found as a function of DRI and the Norwegian Abrasion Value.
BWI is used to predict the rate of wear of percussion drill bits.

Cutter Life Index CLI :
Calculated from the expression $\mathrm{CLI} = 13.84(\mathrm{SJ}/\mathrm{AVS})^{0.3847}$
CLI is one of the input parameters into the NTU-SINTEF method of predicting tunnel boring machine performance.

N.C.B. Cone Indenter Index (N/mm) :

Tests are carried out using a modified (mechanically stiff) version of the NCB Cone Indenter Apparatus, directly measuring the penetration produced by an indenting force of 40N.

The Cone Indenter Index is defined as $\frac{40}{\text{penetration (mm) at 40 N load}}$ (N/mm)

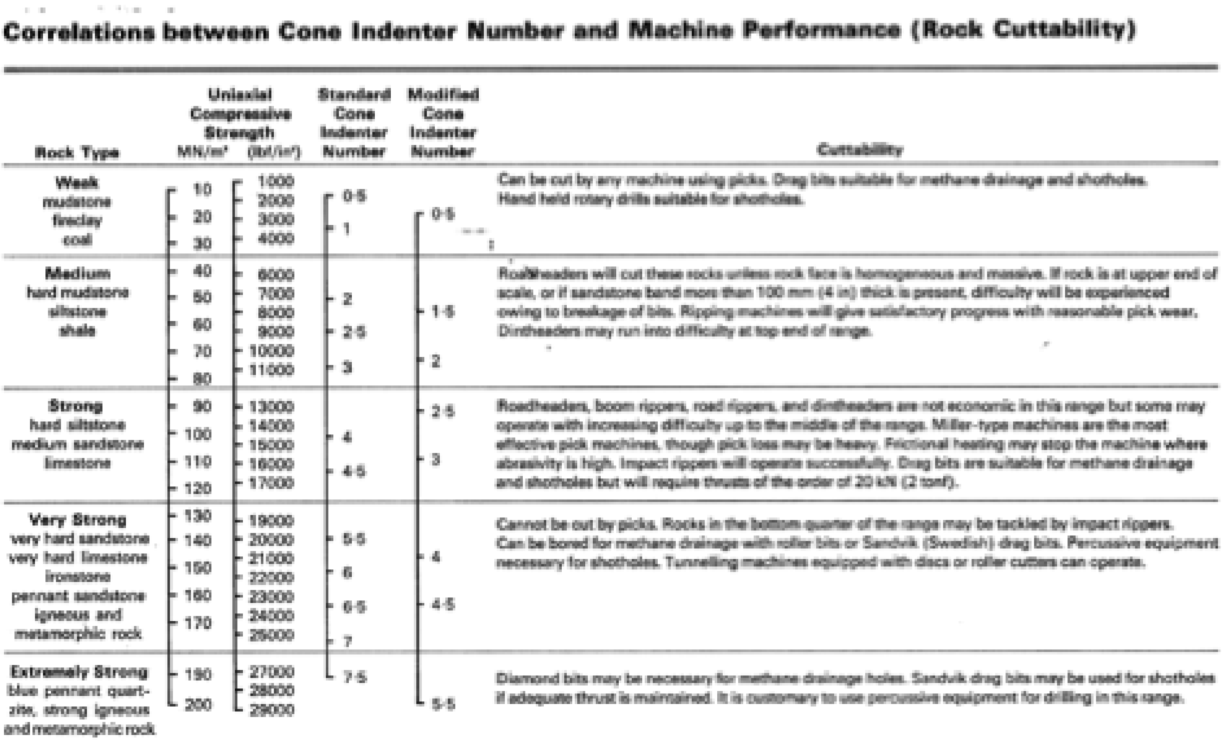
NCB Cone Indenter Hardness I_S :

From consideration of work of Szlavin (1974) the Cone Indenter Hardness is obtained from the Cone Indenter Index using the relationship

$$I_S = \frac{\text{Cone Indenter Index}}{62.5}$$

Note the MRDE's classification of "Rock Type" as a function of Standard Cone Indenter Hardness :

Weak	*0.5 - 1.4*
	(e.g. mudstone, fireclay, coal)
Medium	*1.4 - 3.3*
	(e.g. hard mudstone, siltstone, shale)
Strong	*3.3 - 5.0*
	(e.g. hard siltstone, medium sandstone, limestone)
Very Strong	*5.0 - 7.1*
	(e.g. very hard sandstone, very hard limestone, ironstone, igneous and metamorphic rock)
Extremely Strong	*> 7.1*
	(e.g. quartzite, strong igneous and metamorphic rock)



Correlations between Cone Indenter Number and Machine Performance (Rock Cuttability)

Rock Type	Uniaxial Compressive Strength MN/m²	Uniaxial Compressive Strength (lbf/in²)	Standard Cone Indenter Number	Modified Cone Indenter Number	Cuttability
Weak mudstone fireclay coal	10 20 30	1000 2000 3000 4000	0·5 1	0·5	Can be cut by any machine using picks. Drag bits suitable for methane drainage and shotholes. Hand held rotary drills suitable for shotholes.
Medium hard mudstone siltstone shale	40 50 60 70 80	6000 7000 8000 9000 10000 11000	2 2·5 3	1·5 2	Roadheaders will cut these rocks unless rock face is homogeneous and massive. If rock is at upper end of scale, or if sandstone band more than 100 mm (4 in) thick is present, difficulty will be experienced owing to breakage of bits. Ripping machines will give satisfactory progress with reasonable pick wear. Dintheaders may run into difficulty at top end of range.
Strong hard siltstone medium sandstone limestone	90 100 110 120	13000 14000 15000 16000 17000	4 4·5	2·5 3	Roadheaders, boom rippers, road rippers, and dintheaders are not economic in this range but some may operate with increasing difficulty up to the middle of the range. Miller-type machines are the most effective pick machines, though pick loss may be heavy. Frictional heating may stop the machine where abrasivity is high. Impact rippers will operate successfully. Drag bits are suitable for methane drainage and shotholes but will require thrusts of the order of 20 kN (2 tonf).
Very Strong very hard sandstone very hard limestone ironstone pennant sandstone igneous and metamorphic rock	130 140 150 160 170	19000 20000 21000 22000 23000 24000 25000	5·5 6 6·5 7	4 4·5	Cannot be cut by picks. Rocks in the bottom quarter of the range may be tackled by impact rippers. Can be bored for methane drainage with roller bits or Sandvik (Swedish) drag bits. Percussive equipment necessary for shotholes. Tunnelling machines equipped with discs or roller cutters can operate.
Extremely Strong blue pennant quartzite, strong igneous and metamorphic rock	190 200	27000 28000 29000	7·5	5·5	Diamond bits may be necessary for methane drainage holes. Sandvik drag bits may be used for shotholes if adequate thrust is maintained. It is customary to use percussive equipment for drilling in this range.

(From : MRDE Handbook No. 5 NCB CONE INDENTER)

Morris Drillability, or Punch Penetration Test, or Indentation Hardness Test :
This test, as originally suggested by R. I. Morris, involves pressing a tungsten carbide button indenter vertically down into the horizontal surface of a horizontally-comstrained rock specimen, until a chip is forced out of the rock, while the force and penetration are measured.The force at the onset of chip formation is called the “Threshold Force”, and the Morris Drillability is defined as (the elastic penetration at the onset of chip formation)/(Threshold Force) with units nm/N

The same test results, reported differently, enable calculation of the Brittleness Index (kN/mm), the Handewith Drillability (units pounds/inch), the Dresser Drillability Index (units microinches/lb), and the Tamrock Raise Boring Index RB-i (units microinches/lb.

These tests may be used to predict the rate of raise boring or shaft drilling.
In qualitative terms :

Dresser Drillability (or RB-i)	<10	“Extremely Hard”
	10 – 20	“Hard”
	20 – 30	“Moderately Hard”
	30 – 50	“Moderately Soft”
	>50	“Soft”

Brittleness Index	>40	Very high brittle
	35-40	High brittle
	30-35	Medium brittle
	25-30	Moderate brittle
	20-25	Low brittle
	<20	Ductile

Analysis of results of past testing in this laboratory shows the following distribution of Threshold Force (kN) values:

Lower decile	4.9
Lower quartile	7.5
Median	13.2
Upper quartile	19.4
Upper decile	29.6

C.S.M. Punch Penetration Test :
The Morris Test, in which a button indentor is forced into a confined rock specimen, is also interpreted using the approach of the Colorado School of Mines Earth Mechanics Institute.
Penetration Index δ_i (kN/mm) is the average slope of the Force/Penetration curve from commencement of loading until the final failure, after possibly several chip-forming events.
Compressive Hardness S_c (Mpa)is the average of calculated (Force/Projected chip area) data points; From the measured depth of penetration for a particular value of Force, a chip is assumed to potentially form with a breakout angle of 30° from the point of the indentor up to the rock surface; the plan area of this potential crater is the Projected Chip Area.
The Compressive Hardness is analogous to the bearing capacity of the intact rock substance.
Cutter Penetration divides an assumed value for allowable cutter loading (285kN) by the Penetration Index δ_i to give a value which may be taken as indicative of basic penetration i_o (mm/rev) under a disc cutter.

Stamp Test :
This test was suggested by G. Wijk of Atlas Copco, and is used to calculate the penetration rate of percussion drills and of Tunnel Boring Machines.
A rigid flat-ended cylindrical indentor, with diameter 4mm, made of tungsten carbide, is pressed into the flat end of the rock specimen, which is grouted into a steel confining cylinder with expanding grout.
The force and vertical penetration at which crack initiation and crater formation takes place, F_S and x_s, and the crater volume V are all measured.

The Stamp Test Strength Index σ_{ST} is defined as $F_S /(\pi a^2)$, where a = half the stamp diameter.
The Specific Energy (MJ/m^3) is calculated from the fracture force and the crater volume.
The Brittleness Index is calculated as a function of the Strength Index, the Specific Energy, and the measured Unconfined Compressive Strength of the rock.

CERCHAR Abrasivity :
The width of the wear flat, measured in units of 0.1mm, induced on a sharpened steel needle (HRC = 55) having a 90^{o} conical tip, held with its axis perpendicular to a rock surface, under a load of 7 kg, slowly displaced in a direction parallel to the rock surface for a distance of 10mm, is reported as the CERCHAR Abrasivity Index.

Note the criteria for abrasiveness published by CERCHAR(1986), and modified by Sandvik Mining & Construction (2007) :

0.3 - 0.5	*"not very abrasive"*	<0.5	"not abrasive"
0.5 - 1.0	*"slightly abrasive"*	0.5-1.0	*"little abrasive"*
1.0 - 2.0	*"medium abrasiveness to abrasive"*	1.0-1.3	*"moderately abrasive"*
2.0 - 4.0	*"very abrasive"*	1.3-1.8	*"considerably abrasive"*
4.0 - 6.0	*"extremely abrasive"*	1.8-2.3	*"abrasive"*
6.0 - 7.0	*"quartzitic"*	2.3-3.0	*"very abrasive"*
		3.0-4.5	*"highly abrasive"*
		>4.5	*"extremely abrasive"*

CERCHAR (Dureté*Abrasivity)

CERCHAR Dureté (or Toughness) is not directly measured, but is deduced from published correlations between Uniaxial Compressive Strength and CERCHAR Dureté.
The product of CERCHAR Abrasivity and CERCHAR Dureté is a measure of the difficulty of cutting a rock.
Hughes (1986) was of the opinion that a value of 18 was the desirable upper limit for the use of light-duty roadheaders.
Recent observations indicate that rocks with a value of up to 60 or 80 can be cut by heavy roadheaders.

Schimazek's Wear Index "F"

$F = (V.d.T_o)/100$

Where V = Percentage of hard minerals, standardized against quartz
d = Mean diameter of the contained quartz grains (or of the other dominant hard minerals, multiplied by a reduction factor)
T_o = Brazilian tensile strength

The measured percentage of each mineral is multiplied by a conversion factor based on Mohs hardness e.g.

Mohs hardness	Factor for conversion to quartz
1	0
1.5	0
2	0.0021
2.5	0.015
3	0.036
3.5	0.038
4	0.042
4.5	0.047
5	0.055
5.5	0.16
6	0.31
6.5	0.55
7	1.0

Collective conversion factors can be used e.g.

Hornblende -	0.31
Feldspars -	0.3
Argillaceous minerals -	0.04
Carbonates -	0.03

Correlations by several authors between CERCHAR Abrasivity Index and the Schimazek Wear Index allow the qualitative descriptive terms to be applied to the latter :

CERCHAR Abrasivity Index		Schimazek Wear Index
0.3 - 0.5	*"not very abrasive"*	0.001 – 0.01
0.5 - 1.0	*"slightly abrasive"*	0.01 – 0.1
1.0 - 2.0	*"medium abrasiveness to abrasive"*	0.1 – 0.4
2.0 - 4.0	*"very abrasive"*	0.4 – 1.8
4.0 - 6.0	*"extremely abrasive"*	1.8 – 4.9
6.0 - 7.0	*"quartzitic"*	4.9 – 7.3

Paddle Abrasiveness :
This test was originally developed by the Allis Chalmers Company to describe the abrasiveness of rock fragments, for the specification of materials with which to face the jaws of rock crushers, to resist abrasive wear by the crushed rock. The test was later standardised by the U.S. Bureau of Mines.
Each test material consists of 400 grams of rock fragments, passing through a 3/4 inch (19mm) screen and retained on a 3/8 inch (9.5mm) screen.
A steel paddle is rapidly stirred through the rock fragments for 15 minutes, and the resulting weight loss is measured. The total weight loss caused by 4 test runs, in tenths of a milligram, is reported as the Paddle Abrasiveness.

This is a useful test for indicating whether a crushed and fragmented rock might be expected to cause undue wear to metal wear surfaces, in applications like loaders, scrapers, pipes, etc.

Analysis of past decades' results of testing in this laboratory shows the following distribution of Paddle Abrasiveness values:

Lower decile	37
Lower quartile	88
Median	200
Upper quartile	590
Upper decile	1393

Approximate CERCHAR Abrasivity values deduced from Paddle Abrasiveness:

Compilation of test data indicate a trend line of the form
CERCHAR Abrasivity Index = 0.29*(Paddle Abrasiveness $^{0.40}$)

On this basis, Paddle Abrasiveness values may be assigned the following tentative descriptions:

1-4	*"not very abrasive"*
4-22	*"slightly abrasive"*
22-128	*"medium abrasiveness to abrasive"*
128-738	*"very abrasive"*
738-2056	*"extremely abrasive"*
2056-3036	*"quartzitic"*

The Abroy (LCPC) Abrasimètre

The Abroy abrasimètre is used to investigate the abrasiveness of rocks under varied testing conditions typical of the various types of wear encountered, in accordance with standard NF P 18-579. This abrasimètre was designed at the Laboratoire Centrale des Ponts et Chaussees (LCPC) in France, and the commercial version is sold by IGM.
The equipment has a cylindrical steel receptacle into which is inserted a steel vane (50mm * 25mm * 5mm thick) that rotates in a horizontal plane. A mass of crushed rock is put into the receptacle. The standard mass is 500 g, with a specified material granularity of 4/6.3mm
The stirring action, as the vane rotates through the crushed rock at a speed of 4,500 rpm, subjects the vane to wear, and the rock to breakage.
The abrasivity index is expressed as grams of steel worn off the vane per tonne of material treated, reaching as high as 2500g/t for extremely abrasive rocks.
The breakability index is expressed as the percentage of the original mass which will pass through a 1.6 mm aperture sieve at the conclusion of the test.

Slake Durability
This test determines the resistance offered by a rock sample to weakening and disintegration when subjected to 2 cycles of drying and wetting and physical abrasion in a controlled chemical environment. The test sample comprises ten roughly spherical rock lumps, each with a mass of 40 to 60 grams, to give a total sample mass of 450 to 550 grams. The sample is oven-dried, weighed (measured mass A), then placed in the cylindrical test drum (140mm diameter by 100mm long) which is immersed in a fluid bath to a level 20mm below the drum axis, and rotated at 20 rpm for 10 minutes. The drum is then removed, oven-dried, weighed (measured mass B), then re-immersed, rotated, oven-dried, and weighed (measured mass C). The emptied and dried drum has measured mass D
Slake-durability index $I_{d2} = 100*(C-D)/(A-D)$

Sodium Sulphate Soundness Test
This test determines the resistance of building stone to the forces associated with the crystallization of soluble salts. The test sample comprises either 3 cubes with sides 50mm long, or three 50mm diamond drill cores, each 50mm long. The test solution is prepared by mixing 61.7 grams of anhydrous sodium sulphate with de-ionized water to make 1 litre of solution : equivalent to a 14% solution as the decahydrate. The test sample is oven-dried, weighed (to measure mass m_1), immersed in the solution for 2 hours, then oven-dried for 20 hours and weighed again. This cycle is carried out daily, for 14 more cycles, but with two-day breaks after the 4th, 8th and 12th cycles. The mass of dried intact sample after the 15th cycle is measured (m_2). The disintegrated residue remaining in the test vessel is dried and weighed (m_3).
The percentage mass loss after the 15th cycle (C_{15}) = $100*\delta_m/m_1$
Where δ_m = the larger value of ($m_1 - m_2$) and m_3, in grams

FOR FURTHER INFORMATION CONTACT :
DR. BILL BAMFORD
Telephone : +61 3 9329 2818 (work) or +61 3 9078 9340 (home)
E-mail : bill @bamfordrocks.com.au

CUTTING STRENGTH

Current informed opinion about cutting strength is that it is probably

- proportional to Uniaxial Compressive Strength
- proportional to the square root of a "toughness factor"

This "toughness factor" may be taken as:

1. Inverse of the tan of the Angle of Shearing Resistance (ϕ)

   So : Cutting Strength $\propto C_o/\sqrt{(\tan\phi)}$

2. Inverse of the Brittleness Coefficient:

   $$\frac{\text{Uniaxial Compressive Strength - Tensile Strength}}{\text{Uniaxial Compressive Strength + Tensile Strength}} \qquad (= \sin\phi)$$

   So : Cutting Strength $\propto C_o/\sqrt{(\sin\phi)}$

3. Inverse of the Compressive/Tensile Strength ratio

   So : Cutting Strength $\propto C_o/\sqrt{(C_o/T_o)}$

   $\propto \sqrt{(C_o.T_o)}$

4. Rock Toughness Index

   $$= \frac{1000^*\text{Specific Energy (Strain Energy At Failure)}}{\text{Uniaxial Compressive Strength (MPa)}}$$

   $$= \frac{\text{Specific Energy}}{\text{Uniaxial Compressive Strength}} \quad \frac{(\text{kJ/m}^3)}{(\text{MJ/m}^3)}$$

   So : Cutting Strength $\propto C_o{}^*\sqrt{(\text{R.T.I.})}$

   $\propto C_o{}^*\sqrt{(1000\text{S.E.}/ C_o)}$

   $\propto \sqrt{(C_o{}^*1000\text{S.E.})}$

5. Fracture Toughness

   So : Cutting Strength $\propto C_o{}^*\sqrt{K_{Ic}}$

RATE OF WEAR OF BITS, CUTTERS, OR TOOLS

Current informed opinion about the rate of wear suffered by bits, cutters, or tools while cutting rock is that it is probably

- proportional to Rock Abrasiveness
- proportional to the square root of a "Rock Cutting Strength"

Rock abrasiveness may be measured by 3 different tests :

- The CERCHAR Abrasivity test
- The Norwegian Abrasion Value test
- The Goodrich Wear Number

So,
Wear Rate $\propto \text{C.A.I.}\ \surd[(\ C_o/\surd(\tan\phi)]$
Or $\propto \text{C.A.I.}\ \surd[\ C_o/\surd(\sin\phi)]$
Or $\propto \text{C.A.I.}\ \surd[\surd(C_o.T_o)]$
Or $\propto \text{C.A.I.}\ \surd[\surd(C_o{*}1000\text{S.E.})]$
Or $\propto \text{C.A.I.}\ \surd[\ C_o{*}\surd K_{Ic}]$

Or $\propto \text{N.A.V.}\ \surd[(\ C_o/\surd(\tan\phi)]$
Or $\propto \text{N.A.V.}\ \surd[\ C_o/\surd(\sin\phi)]$
Or $\propto \text{N.A.V.}\ \surd[\surd(C_o.T_o)]$
Or $\propto \text{N.A.V.}\ \surd[\surd(C_o{*}1000\text{S.E.})]$
Or $\propto \text{N.A.V.}\ \surd[\ C_o{*}\surd K_{Ic}]$

Or $\propto \text{G.W.N.}\ \surd[(\ C_o/\surd(\tan\phi)]$
Or $\propto \text{G.W.N.}\ \surd[\ C_o/\surd(\sin\phi)]$
Or $\propto \text{G.W.N.}\ \surd[\surd(C_o.T_o)]$
Or $\propto \text{G.W.N.}\ \surd[\surd(C_o{*}1000\text{S.E.})]$
Or $\propto \text{G.W.N.}\ \surd[\ C_o{*}\surd K_{Ic}]$